UNCLASSIFIED

AD 400 500

Reproduced
by the

ARMED SERVICES TECHNICAL INFORMATION AGENCY
ARLINGTON HALL STATION
ARLINGTON 12, VIRGINIA

UNCLASSIFIED

TRANSLATION

EMPLOYING THE KARMAN METHOD FOR CALCULATING A TURBULENT
BOUNDARY LAYER ON A PLATE IN A GAS FLOW

By

L. G. Loytsanskiy and Yu. V. Lapin

FOREIGN TECHNOLOGY

DIVISION

AIR FORCE SYSTEMS COMMAND

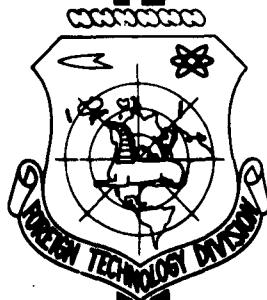### WRIGHT-PATTERSON AIR FORCE BASE

OHIO

UNEDITED ROUGH DRAFT TRANSLATION

EMPLOYING THE KARMAN METHOD FOR CALCULATING A TURBULENT
BOUNDARY LAYER ON A PLATE IN A GAS FLOW

BY: L. G. Loytsanskiy and Yu. V. Lapin

English Pages: 13

SOURCE: Russian Book, "Tekhnicheskaya Gidromekhanika",
Politikhnicheskiy Institute, Nr. 217, 1961,
pp 7-16

S/563-61-0-217

Date 13 Feb. 19 63

Employing the Karman Method for Calculating a Turbulent
Boundary Layer on a Plate in a Gas Flow

by

L. G. Loytsanskiy; Yu. V. Lapin

Introduction

In the theory of turbulent boundary layer in a compressible fluid at greater velo

cities two historical trends have been noticed. The first one of these, discovered by

F.I.Frankl and V.V.Voyshel[1] was based on direct transfer of the/ semiempirical/ thee-

ry by Karman into gas dynamic formulae. Van Drast[2] followed an analogous path, but

with the application of Prandtl formulae.

In reports of the second trend were used known Dorodnitsyn[3] variables and it has

been assumed, that the corresponding formulas of the semiempirical theory of turbulences

should be compiled in these variables [4] .

The answer to the question on which trend offers results more closer to the truth,

could be given only by experiments at greater M-numbers, but by the time of appearance

of both trends experimental data were still lacking. At M-numbers, not much exceeding

one, both trends have given, naturally, results slightly differing from each other.

Only relatively recently, after data have been obtained on the friction resistance of

flat surfaces at greater M-numbers (reaching up to 8 , it became clear, that only the first

trend offers proper results.

We would like to point out, that in the Frankl/Voyshel report the authors stood

along the road of directly generalizing the Karman method, having simplified only

with the assumption of friction stress constancy across the boundary layer. Following

this path, they at first find the form of the profile of velocity in the cross section

of the   layer, then in an ordinary way they find the so-called law of "Resis.
tivity", i.e.relationship between local coefficient of friction and R-number of the
boundary layer   (R-number = Reynolds or Re-number). Then, excluding this Re-number
from equation of the "law of resistivity " and from equation of pulses, they obtain
the sought for relationship between the coefficient of resistance and the Re-number,
formulated by the velocity of an oncoming flow and the abscissa of given point on the
plate.

The degree of approximation , adopted by Frankl and Voyshel, enabled them to per-
sonally carry out calculations but only up to M-numbers,slightly exceeding one. The
change over to greater M-numbers requires, apparently, either an even greater complexity
and besides that a complex calculating method by these authors, or direct application
of numerical methods.

It is shown in this report that when applying the Karman method and assuming fric-
tion stress constancy across the boundary layer, there is a simpler way of formulating
a solution,requiring no introduction of the concept concerning the Reynolds number of the
boundary layer and compilation of the "law of resistivity ". This method considerably
simplifies the investigation of problems pertaining to a turbulent boundary layer in a
gas flow. The use of a simple asymptotic break down allows comparatively easily to
generalize the Karman theory of plate resistance in an incompressible liquid for the
case of a gas flow with greater M-numbers.

1. Turbulent Friction on a Plate in a Flow of Incompressible Liquid.

The integral equation of pulses for a pl.te,around which an incompressible liquid
flows at zero angle of attack, has the form of :

$$\frac{d\delta^{**}}{dx} = \frac{\tau_\omega}{\varrho_\infty U_\infty^2} \qquad (1.1)$$

where x -coordinate,directed along the surface of the plate; U_∞ and ϱ_∞ - velocity
and density of the oncoming flow; τ_ω - friction stress on the wall; δ^{**} - thickness of pulse
loss, expressable by equality:

$$\delta^{**} = \int_0^{\delta} \frac{u}{U_{\infty}} \left(1 - \frac{u}{U_{\infty}}\right) dy. \quad (1.2)$$

In expression (1.2) we substitute the universal coordinates:

$$\varphi = \frac{u}{v_*}, \qquad \eta = \frac{v_* y}{v}; \quad (1.3)$$

here....y —coordinate along the normal to the surface of the plate ; coefficient of kinematic viscosity; $v_* = \sqrt{\frac{\tau_{\omega}}{\varrho}}$ •dynamic velocity.

Then

$$\delta^{**} = \frac{vh}{U_{\infty}} \int_0^{\bar{\varphi}} \frac{\varsigma}{h} \left(1 - \frac{\varsigma}{h}\right) d\eta. \quad (1.4)$$

where

$$h = \frac{U_{\infty}}{v_*}. \quad (1.5)$$

Equality (1.4) can also be written in form of :

$$\delta^{**} = \frac{vh^2}{U_{\infty}} \int_0^{h} \frac{\varsigma}{h} \left(1 - \frac{\varsigma}{h}\right) \dot{\eta} d\left(\frac{\varsigma}{h}\right). \quad (1.6)$$

The value $\dot{\eta} = \frac{d\eta}{d\varphi}$ in the laminary sublayer is equal to one ($\dot{\eta} = 1$). In the turbulent nucleus to determine $\dot{\eta}$ we use the Karman ratie:

$$\tau = \varrho \varkappa^2 \frac{u'^4}{u''^2}, \quad (1.7)$$

where x - turbulence constant, equalling 0.4 (the hatchure sign in the equality (1.7) designates a derivative by y). Adopting a simplified assumption about the friction stress constancy across layer $\gamma = \gamma_{\omega}$ and transition in (1.7) toward universal coor dinates (1.3) we will obtain:

$$\frac{\varsigma''}{\varsigma'^2} = -\varkappa. \quad (1.8)$$

The minus sign appears after ext action of the root in connection with the fact, that $\varphi'' < 0$. In equation (1.8) we change the argument and function places, then we obtain an equation:

$$\frac{\ddot{\eta}}{\dot{\eta}} = \varkappa, \quad (1.9)$$

from which it is evident that

$$\dot{\eta} = C_1 e^{\varkappa \varphi}. \quad (1.10)$$

If we should designate by the letter f the magnitude of the derivative $\frac{d\varphi}{d\eta}$ on the boundary of the laminary sublayer $\eta = \eta_{\lambda} = a$ from the side of the turbulent nucle us, and writing

$$f = \left(\frac{d\varphi}{d\eta}\right)_{\eta=\eta_1-0} = \varphi'(a-0) = \frac{1}{\kappa a}. \quad (1.11)$$

then equation (1.10) will acquire the form of

$$\eta = \frac{e^{-\kappa a}}{f} e^{\kappa \varsigma}, \quad (1.12)$$

where a - turbulence constant, equalling 11.5.

Returning to equation (1.6) we want to point out, that, strictly speaking, the integral in the right part of this equation should have been broken down into two sections: $0 \leq \varphi \leq a$ and $a \leq \varphi \leq h$ and place in each one of these a value η, namely $\eta = 1$ in the first and according to equation (1.12) - in the second. But in view of the relatively thin (relative thinness) of the laminary sublayer it is possible to bypass the first section, extending the second one to the wall. As is shown by calculations in first approximation, at greater h no noticeable difference is obtained.

In this way, eliminating η from equation (1.6) through (1.12) we will obtain the following expression for pulse loss thickness:

$$\delta^{**} = \frac{\nu h \cdot e^{-\kappa a}}{f U_{\infty}} \int_0^1 \overline{u}(1-\overline{u}) e^{\kappa h \overline{u}} d\overline{u}, \quad (1.13)$$

where $\overline{u} = \frac{\varphi}{h}$.

The value κh, as is evident from its determination, considerably exceeds one. In these cases it is convenient to use the following general idea of the integral (containing the exponential function), which is obtained as result of integrating by parts:

$$\int e^{\kappa h \psi} f(\psi) d\psi = \frac{e^{\kappa h \psi}}{\kappa h} \left[f(\psi) - \frac{f'(\psi)}{\kappa h} - \frac{f''(\psi)}{\kappa^2 h^2} - \cdots \right]. \quad (1.14)$$

[1]n this case, when the series in square parentheses remains undisrupted, formula (1.14) can be considered as an asymptotic series, expressing the value of the integral at greater h.

Making in (1.13) an integration with the aid of (1.14) we will obtain an expression for pulse less thickness :

$$\delta^{**} = \frac{e^{-\kappa a}\nu}{f \kappa^2 U_{\infty}} e^{\kappa h}\left(1 - \frac{2}{\kappa h}\right). \quad (1.15)$$

We will also obtain an expression for the form-parameter H;

$$H = \frac{\delta^*}{\delta^{**}}. \quad (1.16)$$

where δ^* - thickness of squeezing out the boundary layer, expressed by equation

$$\delta^* = \int_0^{\cdot}\left(1 - \frac{u}{U_\infty}\right)dy. \quad (1.17)$$

Using a method analogous to the one shown above, it is possible to find the following expression for the thickness of squeeze out:

$$\delta^* = \frac{e^{-\kappa a}v}{\int x \cdot U_\infty} e^{\kappa h}. \quad (1.18)$$

Compiling the ratio δ^*/δ^{**} according to equations (1.18) and (1.15) we will obtain a term for H:

$$H = \frac{1}{1 - \frac{2}{\kappa h}}. \quad (1.19)$$

which at Re numbers 10^6 - 10^7 gives a value H = 1.3, well conforming with the experiment.

Turning to expression (1.15), we want to mention, that in view of the exponential dependence of pulse loss thickness upon h, when it is being calculated it is possible to disregard the second component in comparison with the first one. As was shown by calculations, at greater values h the error obtainable when calculating the friction coefficient, is insignificant at such an approximation.

Having accepted for pulse loss thickness such an approximation, we substitute its expression in the integral formula of pulses (1.1). After simple transform we will obtain the following equation:

$$\frac{e^{-\kappa a}}{\int x^2} h^2 de^{\kappa h} = dR_\lambda. \quad 1.20$$

where

$$R_x = \frac{U_\infty x}{v}. \quad (1.20a)$$

Taking the integral from both parts of this equation and utilizing the boundary condition $R_x = 0$ at h = 0, we will have in this approximation

$$\frac{e^{-\kappa a}}{\int x^2} h^2 e^{\kappa h} = R_x. \quad (1.21)$$

Logarithming (1.21) and turning away from variable h to ef according to equation

$$h = \sqrt{\frac{2}{c_f}}. \quad (1.22)$$

we will obtain the known Karman formula

$$\frac{1}{\sqrt{c_f}} = A + B \lg (R \sqrt{c_f}) \cdot \qquad (1.23)$$

The just now explained order of deriving the Karman formula is not in itself impor-
tant, but only as an explanation to a more complex derivation of Karman formula general-
ization for the case of a gas flow.

2. Turbulent boundary layer in the flow of a compressible gas

We will confine ourselves to the case when the Prandtl number equals one (Pr=1)
and will discuss the flow around a plate, as thermally insulated ("lamellar thermo-
meter") and with discharge of heat as well.

We will assume, that in any given section of the boundary layer is valid the fol-
lowing known ratio between the absolute temperature of the gas T and its longitudinal
velocity u

$$\frac{T}{T_w} = 1 - \omega \left(\frac{\varsigma}{h}\right) - \gamma \left(\frac{\varphi}{h}\right)^2 .$$

where

$$\omega = 1 - \frac{Tt}{Tw} \quad (\omega > 0 \text{ when the surface is heated}$$

$$\text{and } \omega < 0 \text{ when cooled}); \text{ gamma} = \frac{k-1}{2} M_\omega^2 \frac{Tt}{Tw} ;$$

$Tt = T_f (1 + \frac{k-1}{2} M_\omega^2)$ - temperature of the thermometer, equalling the temper-
ature of plate surface Tw in the absence of heat transfer ($\omega = 0$).

Noting, that at a pressure constancy over the profile of the layer takes place an

equality $$\frac{\varrho}{\varrho_w} = \frac{T_w}{T} = \frac{1}{1 - \omega \left(\frac{\varphi}{h}\right) - \gamma \left(\frac{\varsigma}{h}\right)^2} . \qquad (2.2)$$

we will compile analogous with (1.8) the basic Karman formula, which in the presence
of compressibility of the medium will have the form of:

$$\frac{\varphi'}{\varphi'^2} = - \frac{x}{\sqrt{1 - \omega \left(\frac{\varphi}{h}\right) - \gamma \left(\frac{\varsigma}{h}\right)^2}} . \qquad (2.3)$$

Substituting, as before in par.1. the argument and function places, we will obtain
instead of (2.3) an equation

$$\frac{\ddot{\eta}}{\eta} = \frac{x}{\sqrt{1 - \omega \left(\frac{\varphi}{h}\right) - \gamma \left(\frac{\varphi}{h}\right)^2}} . \qquad (2.4)$$

as first integral in which is served by

$$\dot{\eta} = C \exp\left(\frac{xh}{\sqrt{\gamma}} \text{ arc sin } \frac{\sqrt{\gamma}\frac{u}{h} - \frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1 + \frac{\omega^2}{4\gamma}}} \right). \quad (2.5)$$

To determine the constants C it is necessary to deal with the value of the derivative $\frac{d\varphi}{d\eta}$ on the boundary of the laminary sublayer from the side of the turbulent nucleus. A simpler assumption appears to be the requirement, that this value should be of the very same magnitude $f = \frac{1}{xa}$, as in the case of low velocities; in other words, it is assumed, that φ (a+0) does not depend upon the gamma and omega coefficients. Such an assumption is equivalent to disregarding a change in density along the length of the laminary sublayer; it leads to the following result:

$$\eta = \frac{1}{f} \exp\left[\frac{xh}{\sqrt{\gamma}}\left(\text{arc sin } \frac{\sqrt{\gamma}\,\bar{u}}{\sqrt{1 + \frac{\omega^2}{4\gamma}}} - \text{arc sin } \frac{\sqrt{\gamma}\bar{u} + \sqrt{\gamma}}{\sqrt{1 + \frac{\omega^2}{4\gamma}}} \right) \right]. \quad (2.6)$$

One can be convinced easily, that at omega = gamma= 0 the right part of equation (2.6) coincides with the corresponding equation (1.12) for incompressible fluid.

After this we will take up the thickness of pulse loss of the boundary layer. In case of a compressible gas this value is expressed by equation:

$$\delta^{**} = \int_0^\infty \frac{\varrho u}{\varrho_\infty U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy. \quad (2.7)$$

Using equations (2.2) and (2.6) and making transforms, analogous as in previous paragraph, we will obtain the following expression for the thickness of pulse loss:

$$\delta^{**} = \frac{(1 - \omega - \gamma) h^2 v_{\omega}}{fU_\infty} \int_0^1 \frac{\bar{u}(1 - \bar{u})}{1 - \omega\bar{u} - \gamma u^2} \times$$

$$\times \exp\left[\frac{xh}{\sqrt{\gamma}}\left(\text{arc sin } \frac{\sqrt{\gamma}\,\bar{u} - \frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1 + \frac{\omega^2}{4\gamma}}} - \text{arc sin } \frac{\sqrt{\gamma}\,\bar{a} + \frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1 + \frac{\omega^2}{4\gamma}}} \right) \right]. \quad (2.8)$$

Introducing a new variable

$$\psi = \frac{1}{\sqrt{\gamma}} \text{ arc sin } \frac{\sqrt{\gamma}\,\bar{u} + \frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1 + \frac{\omega^2}{4\gamma}}}. \quad (2.9)$$

we transform (2.8) into form of

$$\delta^{**} = \frac{(1-\omega-\gamma)v_-h^2\,]\sqrt{1-\frac{\omega^2}{4\gamma}}}{lC_-\gamma} \int \frac{\exp[x h(v-v_-)]}{\cos\sqrt{\gamma}\,v} $$

$$\times \left(\sin\sqrt{\gamma}\,\psi - \sin\sqrt{\gamma}\,v_- \;\;\sin\sqrt{\gamma}\,\psi_- - \sin\sqrt{\gamma}\,\psi\right)d\psi. \qquad (2.10)$$

where

$$\psi_- = \frac{1}{\sqrt{\gamma}}\,\text{arc}\sin\frac{\sqrt{\gamma}+\frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1-\frac{\omega^2}{4\gamma}}};$$

$$\psi_\omega = \frac{1}{\sqrt{\gamma}}\,\text{arc}\sin\frac{\frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1+\frac{\omega^2}{4\gamma}}};$$

$$\psi_a = \frac{1}{\sqrt{\gamma}}\,\text{arc}\sin\frac{\sqrt{\gamma}\,a-\frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1+\frac{\omega^2}{4\gamma}}}.$$

(2.11)

To calculate the determined integral, situated in the right side of

equation (2.10), we will again assume that the method of symtotic analysis,

the same as in the previous paragraph. Intergrating with an accuracy to the

member, containing $x^3 h^3$ in the denominator and substituting approximation ψ_a

in form of $\qquad \psi_a = \psi_\omega - \frac{a}{h}. \qquad 2.12$

we will obtain the following expression for pulse thickness loss

$$\delta^{**} = \frac{e^{-xa}(1-\omega-\gamma)v_\omega}{4\pi^2 U_\infty}e^{xh(v_\omega-v_-)}\left(1-\frac{2-1.5\omega-\gamma}{xh\sqrt{1-\omega-\gamma}}\right) \qquad (2.13)$$

In an analogous way it is possible to obtain the expression for the

thickness of displacement $\qquad \delta^* = \frac{e^{-xa}(1-\gamma)v_\omega}{lx^2U_\infty}e^{xh(v_\omega-v_-)}\left[1-\frac{\gamma\left(3-1.5\omega-\gamma-\frac{\omega}{2\gamma}\right)}{xh(1-\gamma)\sqrt{1-\omega-\gamma}}\right] \qquad (2.14)$

Compiling, according to equations (2.14) and (2.13), the ratio $\delta^*/\delta^{**} = H$,

we will find:

$$H = \frac{(1-\gamma)\left[1-\frac{\gamma\left(3-1.5\omega-\gamma-\frac{\omega}{2\gamma}\right)}{xh(1-\gamma)\sqrt{1-\omega-\gamma}}\right]}{(1-\omega-\gamma)\left[1-\frac{2-1.5\omega-\gamma}{xh\sqrt{1-\omega-\gamma}}\right]}. \qquad (1.15)$$

In a specific case of flow of an incompressible fluid (gamma=0) and in the

absence of heat transfer (omega = 0) we obtain a term for H, perfectly

coinciding with the corresponding equation of the previous paragraph (1.19).

Having assumed, as in previous paragraph, for pluse loss thickness a more

cruder approximation (disregarding the second component in eq. (2.13) we will

substitute this

equation

$$\delta^{**} = \frac{e^{-\varkappa a} v_{\omega}(1 - \omega - \gamma)}{j\varkappa^2 U_{\infty}} e^{\varkappa h(\Psi_{\infty} - \Psi_{\omega})}$$

into an integral equation of pulses, coinciding in form with (1.1). After simple trans_
forms we will obtain an equation:

$$\frac{e^{-\varkappa a}}{j\varkappa^2} h^2 de^{\varkappa h(\Psi_{\infty} - \Psi_{\omega})} = dR_{x\omega}. \qquad (2.17)$$

where

$$R_{x\omega} = \frac{U_{\infty} x}{v_{\omega}}. \qquad (2.17a)$$

Having taken the integral from both parts of this equation and using boundary
condition $R_{x\omega} = 0$ at $h = 0$, we will obtain in this approximation

$$\frac{e^{-\varkappa a}}{j\varkappa^2} h^2 e^{\varkappa h(\Psi_{\infty} - \Psi_{\omega})} = R_{x\omega}. \qquad (2.18)$$

Equation (2.18) transforms at the limit into a corresponding expression for
incompressible fluid $\qquad \frac{e^{-\varkappa a}}{j\varkappa^2} h_{\circ} e^{\varkappa h_{\circ}} = R_{x0}. \qquad (2.19)$

Next we will separate both parts of equation (2.18) into conformal two parts in
equation (219), and then we will have

$$\left(\frac{h}{h_{\circ}}\right)^2 e^{\varkappa h(\Psi_{\infty} - \Psi_{\omega}) - \varkappa h_{\circ}} = \frac{\varrho_{\omega} \mu_{\infty}}{\varrho_{\infty} \mu_{\omega}}. \qquad (2.20)$$

Changing over in (2.20) from h to c_f and logarithming, we will obtain:

$$-\lg \frac{c_{f0}}{c_f} + \frac{\varkappa \sqrt{2} \lg e}{c_{f0}} \left[\frac{(\Psi_{\infty} - \Psi_{\omega}) \sqrt{1 - \omega - \gamma}}{\sqrt{\frac{c_f}{c_{f0}}}} - 1 \right] = \lg \left(\frac{\mu_{\infty}}{\mu_{\omega}}\right). \qquad (2.21)$$

Accepting for c_{f0} a gradual dependence upon the number R_{x0}

$$c_{f0} = 0.0263 R_{x0}^{-1/7} \qquad (2.22)$$

and introducing designations

$$F = \frac{0.25}{\sqrt{c_{f0}}} = 1.55 R_{x0}^{1/14};$$

$$G = \lg\left(\frac{\mu_{\infty}}{\mu_{\omega}}\right);$$

$$K = \sqrt{\frac{1 - \omega - \gamma}{\gamma}} \left(\arcsin \frac{\sqrt{\gamma} + \frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1 + \frac{\omega^2}{4\gamma}}} - \arcsin \frac{\frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1 + \frac{\omega^2}{4\gamma}}} \right). \qquad (2.23)$$

we will obtain a dependence of the friction coefficient upon the parameters of the

uneoming flow and conditions on the wall in the following unclear form:

$$-\lg\left(\frac{c_I}{c_{I0}}\right) \div FK\Big] \sqrt{\frac{c_{I0}}{c_I}} = F \div G. \qquad (2.24)$$

Equation (2.24) can be reduced to a transcedental equation with one parameter in form of :

$$\lg N \div N = L, \qquad (2.25)$$

where

$$\begin{aligned} L &= \lg\frac{FK}{2} \div \frac{F \div G}{2}; \\ N &= \frac{FK}{2\sqrt{\frac{c_I}{c_{I0}}}}. \end{aligned} \Bigg\} \qquad (2.26)$$

In this way, the expression for the friction coefficient can be written:

$$\frac{c_I}{c_{I0}} = \frac{F^2 K^2}{4N^2}. \qquad (2.27)$$

where F an K - known functions, and N is determined from solution of equation (2.25).

As was shown by calculations, equation (2.25) can be replaced by an approximated one, by a simpler equation:

$$N = 0.123 \div 0.820\,L, \qquad (2.28)$$

which appears to be sufficiently accurate in a wide range of changes of all parameters. In this case equation (2.27) acquires the form of:

$$\frac{c_I}{c_{I0}} = \frac{F^2 K^2}{4\left[0.123 \div 0.820\left(\lg\frac{FK}{2} + \frac{F - G}{2}\right)\right]^2}. \qquad (2.29)$$

To determine value G it is necessary to accept a law of change in viscosity due to temperature. In the role of such a law can be accepted either the Sezerlend formula, or the gradual viscosity dependence upon temperature. In the latter case the expression for G has the form of :

$$G = n \lg\left(\frac{T_\infty}{T_w}\right). \qquad (2.30)$$

where n- index of the d gree in the $\mu \sim T^n$ law. It would be interesting to compare the described method with the Van-Drayst method, which, as is known, is based on the transfer of the Prandtl formula into gas dynamics. After the Van-Drayst formula is brought to form (2.24) it becomes explained, that the difference lies in the value G, which

during the adoption of the gradual law of change in viscosity is expressed by equation:

$$G = \left(n + \frac{1}{2}\right) \lg \left(\frac{T_\infty}{T_c}\right). \quad \textit{2.30a}$$
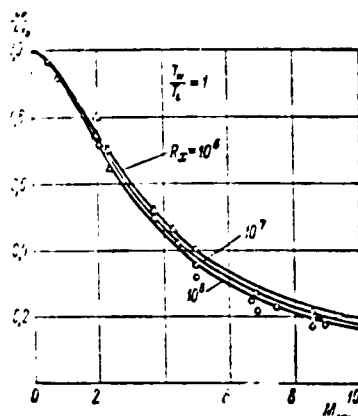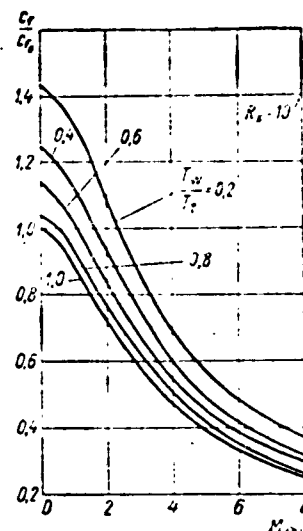

Fig.1.

Fig.2

Upon the examination of equation (2.29) it is revealed that at greater values of Re-number the magnitude of the ratio c_f/c_{f_0} depends very weakly upon the Re-number. The truth is, directing the Re-number into infinity ($R_x{}^0 \rightarrow \infty$) or F toward infinity, we obtain the following specific formula for the ratio c_f/c_{f_0}:

$$\frac{c_f}{c_{f_0}} = K^2. \quad \left(\textit{2.30b}\right)$$

This formula may appear to be useful when making approximate calculations of friction resistance at greater Re-numbers.

According to the described method (formula (2.29((the local friction coefficient was calculated. The results of calculation for the case of absence of heat transfer (T_w/T_t = 1) and comparison of same with experimental data of different authors[5]-[8] are shown in fig.1.Three curves on this drawing correspond to three different values of Re-number ($10^6, 10^7, 10^8$).

As is evident from the drawing, the calculated and experimental data are in excellent conformity all the way up to greater M numbers.

The effect of the temperature factor on the friction coefficient is shown in fig.2.

We shall examine two available practical values of partial cases.

1) Plate Heat Insulated

In this case the temperature of the wall T_w is equal to the temperature of deceleration:

$$T_\_ \quad T_\sim \left(1 - \frac{k-1}{2} M_\cdot^2\right). \quad (2.31)$$

and the values omega and gamma res ectively are equal to

$$\omega \quad 0, \quad \gamma \quad \frac{\frac{k-1}{2} M_\cdot^2}{1 - \frac{k-1}{2} M_\infty^2}. \quad (2.32)$$

Function K determining the process will be:

$$K = \left| \frac{1-\gamma}{\gamma} \text{ arc sin } \right| \overline{\gamma}. \quad (2.33)$$

2) Plate with heating or cooling at small velocities

In this case, guiding gamma → 0, we will have:

$$K \_ \frac{2(\sqrt{1-\omega} - \omega - 1)}{\omega}. \quad (2.34)$$

In the special case of flow of an incompressible fluid in the absence of heat transfer (omega = gamma = 0) the value K = 1.

Literature

1. Frankl',F.I.Voyshel',V.V;Transactions of TSAGI,No.321, 1937
2. Van-Drayst Ye.K.Me hanics Foreign Literature,collection of translations No.1.1952
3. Dorodnitsyn,A.A. PMM,vol.IX,1945,vol.X. 1946
4. Kalikhman, L. E. PMM T. IKH, 1945, T. KH, 1946
5. Chapman D. and Kester R., JAS, N 7, vol. 20, 1953
6. Coles D., JAS, No 10, 1952, vol. 19
7. Hill F. K., JAS, № 1, 1953, vol. 20
8. Lobb R. K., Winkler E. M., Persch J., JAS, № 1, 1955, vol. 22

DISTRIBUTION LIST

DEPARTMENT OF DEFENSE	Nr. Copies	MAJOR AIR COMMANDS	Nr. Copies
		AFSC	
		SCFDD	1
		ASTIA	25
HEADQUARTERS USAF		TDBTL	5
		TDBDP	5
AFCIN-3D2	1	AEDC (AEY)	1
ARL (ARB)	1	SSD (SSF)	2
		AFFTC (FTY)	1
		AFSWC (SWF)	1
OTHER AGENCIES			
CIA	1		
NSA	6		
DIA	9		
AID	2		
OTS	2		
AEC	2		
PWS	1		
NASA	1		
ARMY	3		
NAVY	3		
RAND	1		